

Probing quantum vacuum geometrical effects with cold atoms

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The Casimir-Polder force

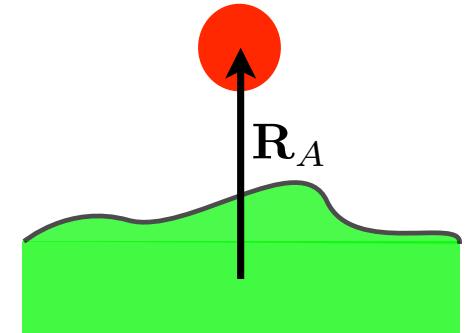


vdW - CP interaction

Casimir and Polder (1948)

The interaction energy between a ground-state atom and a surface is given by

$$U_{\text{CP}}(\mathbf{R}_A) = \frac{\hbar}{c^2 \epsilon_0} \int_0^\infty \frac{d\xi}{2\pi} \xi^2 \alpha(i\xi) \text{Tr } \mathbf{G}(\mathbf{R}_A, \mathbf{R}_A, i\xi)$$



Atomic polarizability: $\alpha(\omega) = \lim_{\epsilon \rightarrow 0} \frac{2}{3\hbar} \sum_k \frac{\omega_{k0} |\mathbf{d}_{0k}|^2}{\omega_{k0}^2 - \omega^2 - i\omega\epsilon}$

Scattering Green tensor: $\left(\nabla \times \nabla \times -\frac{\omega^2}{c^2} \epsilon(\mathbf{r}, \omega) \right) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}')$

Eg: Ground-state atom near planar surface @ T=0

Non-retarded (vdW) limit $z_A \ll \lambda_A$

$$U_{\text{vdW}}(z_A) = -\frac{\hbar}{8\pi\epsilon_0} \frac{1}{z_A^3} \int_0^\infty \frac{d\xi}{2\pi} \alpha(i\xi) \frac{\epsilon(i\xi) - 1}{\epsilon(i\xi) + 1}$$

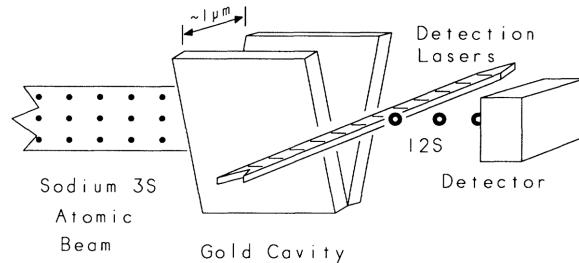
Retarded (CP) limit $z_A \gg \lambda_A$

$$U_{\text{CP}}(z_A) = -\frac{3\hbar c \alpha(0)}{8\pi} \frac{1}{z_A^4} \frac{\epsilon_0 - 1}{\epsilon_0 + 1} \phi(\epsilon_0)$$

Modern CP experiments

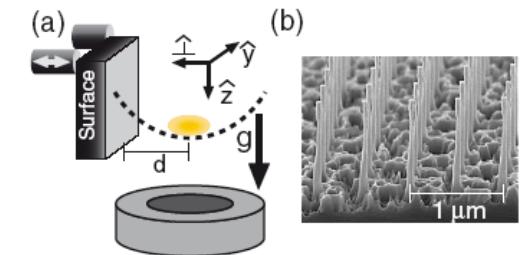
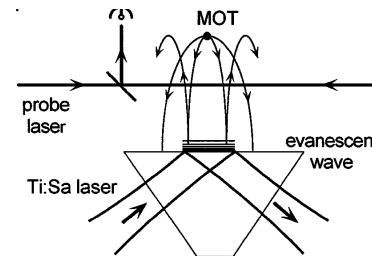


Deflection of atoms



Hinds et al (1993)

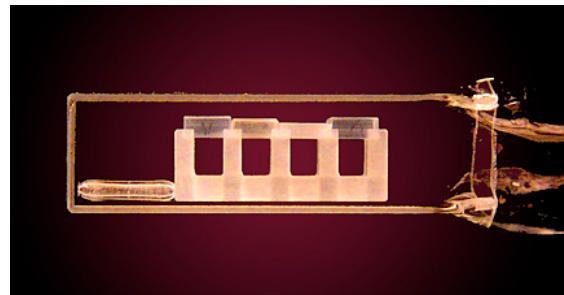
Quantum reflection



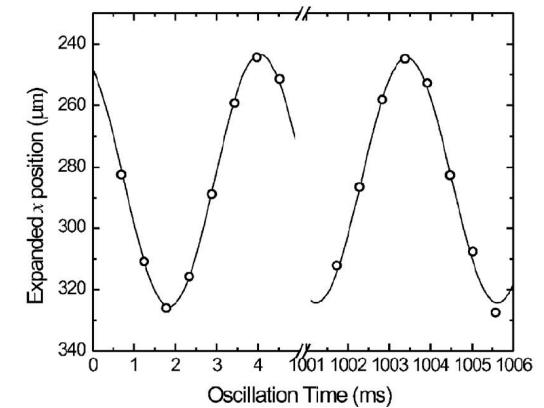
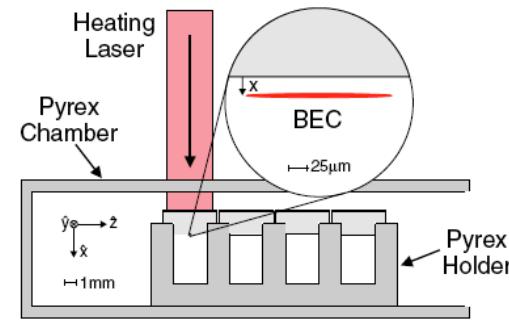
Aspect et al (1996)

Ketterle et al (2006)

Bose-Einstein condensate oscillator

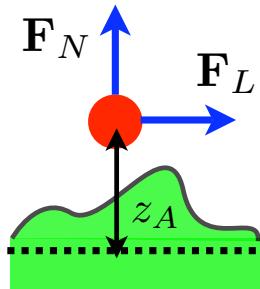


Cornell et al (2007)



$$\gamma_x \equiv \frac{\omega_x - \omega'_x}{\omega_x} \simeq -\frac{1}{2\omega_x^2 m} \partial_x^2 U^*$$

CP within scattering theory



$$U_{\text{CP}} = U_{\text{CP}}^{(0)}(z_A) + U_{\text{CP}}^{(1)}(z_A, x_A)$$



Normal CP force:

$$U_{\text{CP}}^{(0)}(z_A) = \frac{\hbar}{c^2 \epsilon_0} \int_0^\infty \frac{d\xi}{2\pi} \xi^2 \alpha(i\xi) \int \frac{d^2 \mathbf{k}}{(2\pi)^2} \frac{1}{2\kappa} \sum_p \hat{\epsilon}_p^+ \cdot \hat{\epsilon}_p^- r_p(\mathbf{k}, \xi) e^{-2\kappa z_A}$$



Lateral CP force:

$$U_{\text{CP}}^{(1)}(z_A, x_A) = \int \frac{d^2 \mathbf{k}}{(2\pi)^2} e^{i\mathbf{k} \cdot \mathbf{r}_A} g(\mathbf{k}, z_A) H(\mathbf{k})$$

Response function g:

$$g(\mathbf{k}, z_A) = \frac{\hbar}{c^2 \epsilon_0} \int_0^\infty \frac{d\xi}{2\pi} \xi^2 \alpha(i\xi) \int \frac{d^2 \mathbf{k}'}{(2\pi)^2} a_{\mathbf{k}', \mathbf{k}' - \mathbf{k}}(z_A, \xi)$$

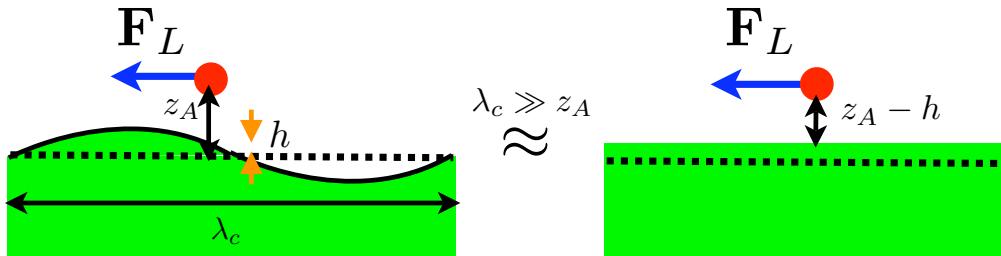
$$a_{\mathbf{k}', \mathbf{k}''} = \sum_{p', p''} \hat{\epsilon}_{p'}^+ \cdot \hat{\epsilon}_{p''}^- \frac{e^{-(\kappa' + \kappa'') z_A}}{2\kappa''} R_{p', p''}(\mathbf{k}', \mathbf{k}'')$$

Our approach is perturbative in $h(x, y)$, which should be the smallest length scale in the problem $h \ll z_A, \lambda_c, \lambda_A, \lambda_0$

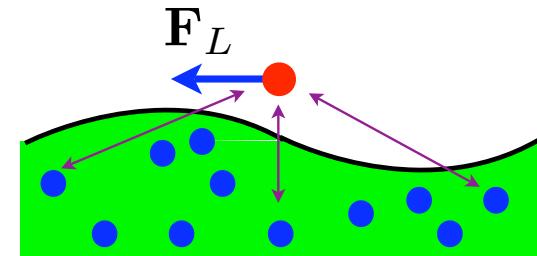
Geometry effects: Lateral CP



Proximity Force Approximation (PFA)

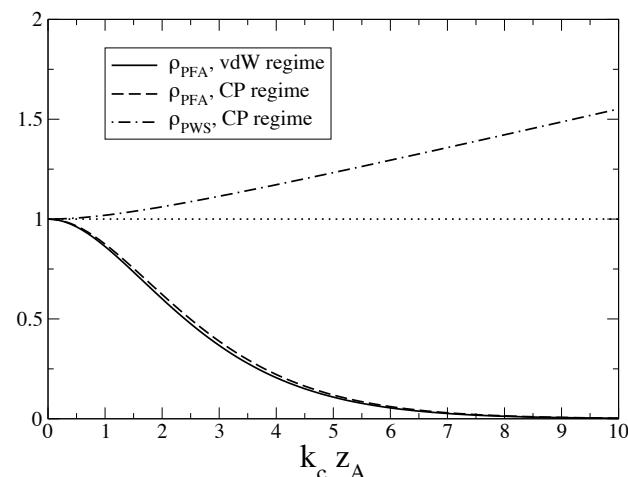


Pair-wise Summation (PWS)



Deviations from PFA and PWS

$$\rho_{\text{PFA}} = \frac{g(k_c, z_A)}{g(0, z_A)} \quad \rho_{\text{PWS}} \equiv \frac{g(k_c, z_A)}{g_{\text{PWS}}(k_c, z_A)}$$



Example:

atom-surface distance $z_A = 2\mu\text{m} \gg \lambda_A$
 corrugation wavelength $\lambda_c = 3.5\mu\text{m}$

→ $\rho_{\text{PFA}} \approx 30\%$ $\rho_{\text{PWS}} \approx 115\%$

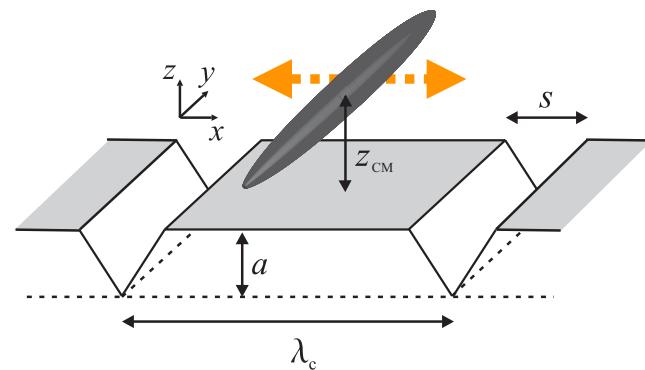
PFA largely overestimates the lateral CP force
 PWS underestimates the lateral CP force

A BEC quantum vacuum sensor



BEC oscillator

In order to measure the lateral component $U_{\text{CP}}^{(1)}(x, z)$, a cigar-shaped BEC could be trapped parallel to the corrugation lines, and the **lateral dipolar oscillation** measured as a function of time



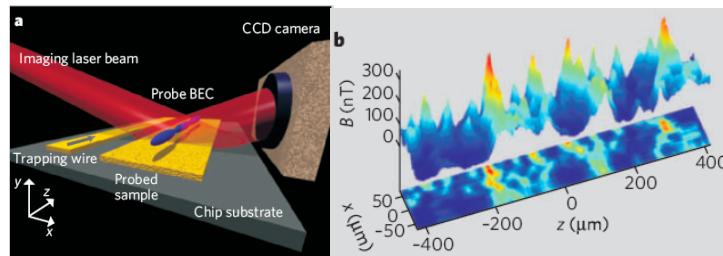
$$V(\mathbf{r}) = V_{\text{ho}}(\mathbf{r}) + U_{\text{CP}}(\mathbf{r})$$

$$V_{\text{ho}}(\mathbf{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \quad \omega_y \ll \omega_x = \omega_z$$

Lateral frequency shift:

$$\omega_{x,\text{CM}}^2 = \omega_x^2 + \frac{1}{m} \int dx dz n_0(x, z) \frac{\partial^2}{\partial x^2} U_{\text{CP}}^{(1)}(x, z)$$

Density variations of a BEC above an atom chip



Density modulation:

$$V_{\text{ho}}(x) + U_{\text{CP}}(x) = -\hbar \omega_x \sqrt{1 + 4a_{\text{scat}} n_{1d}(x)}$$

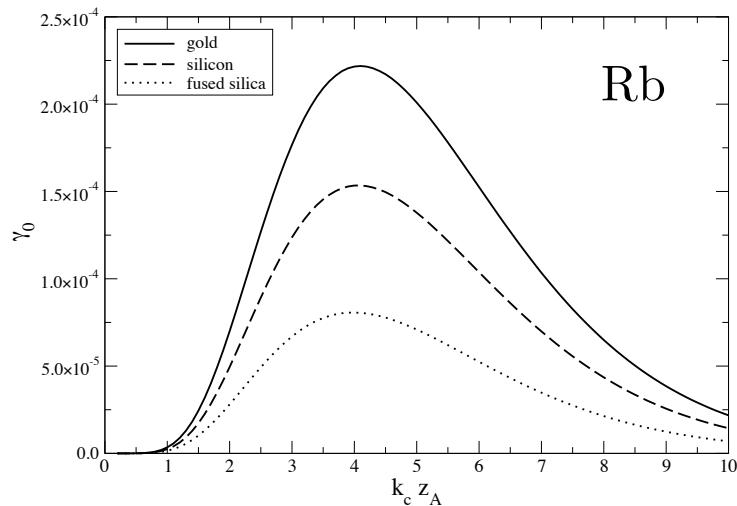
Single-atom/BEC frequency shift



Relative frequency shift:

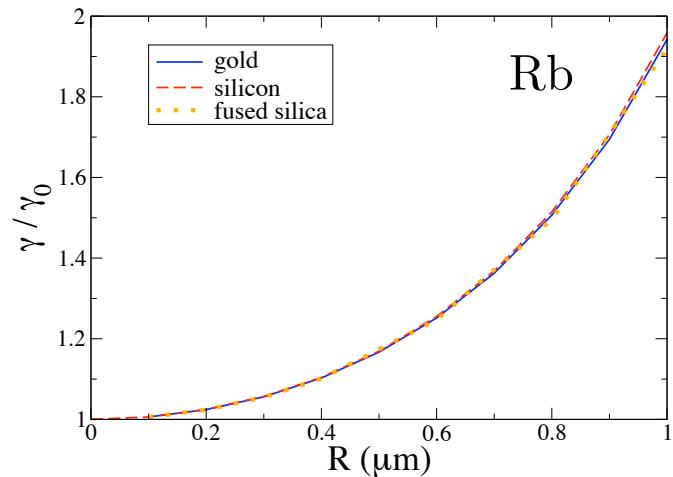
$$\gamma_0 \equiv \frac{\omega_{x,\text{CM}} - \omega_x}{\omega_x}$$

Single-atom lateral freq. shift



$$\begin{aligned} \omega_x / 2\pi &= 229 \text{ Hz} \\ z_{\text{CM}} &= 2 \mu\text{m} \\ \lambda_c &= 4 \mu\text{m} \\ a &= 250 \text{ nm} \\ s &= \lambda_c / 2 \end{aligned}$$

Single-atom / BEC comparison



Given the **reported sensitivity** $\gamma = 10^{-5} - 10^{-4}$ for relative frequency shifts from E. Cornell's experiment, we expect that **beyond-PFA lateral CP forces on a BEC above a plateau of a periodically grooved silicon surface should be detectable for distances $z_{\text{CM}} < 3 \mu\text{m}$, groove period $\lambda_c = 4 \mu\text{m}$, groove amplitude $a = 250 \text{ nm}$, and a BEC radius of, say, $R \approx 1 \mu\text{m}$**

Summary



- Novel cold atoms techniques open a promising way of investigating nontrivial geometrical effects on quantum vacuum
- Important feature of atoms: they can be used as local probes of quantum vacuum fluctuations
- We predict large deviations from PFA and PWS for the lateral Casimir-Polder force on an atom above a corrugated surface
- Non-trivial, beyond-PFA effects should be measurable using a BEC as a vacuum field sensor with available technology

For more details see: Dalvit, Maia Neto, Lambrecht, and Reynaud, PRL 100, 040405 (2008) and arXiv:0710.5249